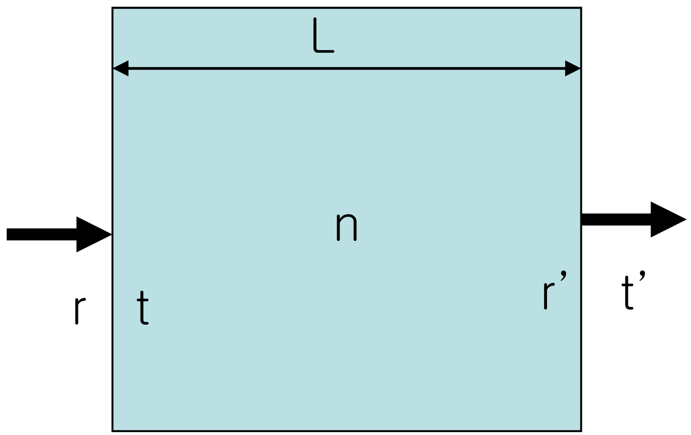
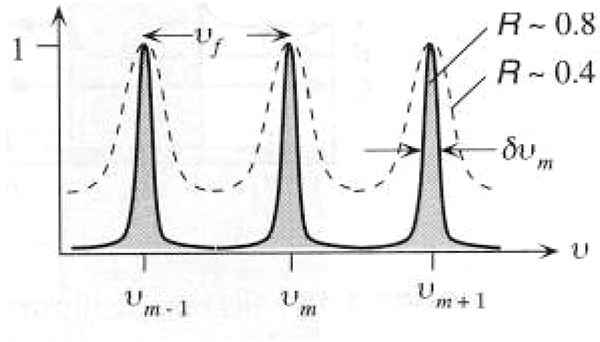
Lect. 8: Interferometers

Fabry-Perot Interferometer





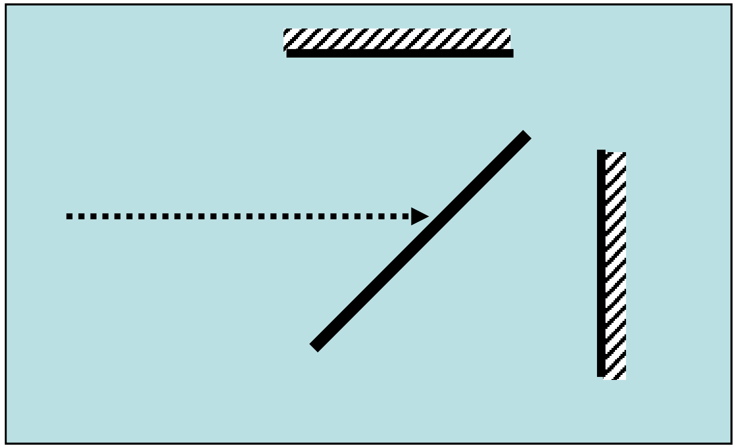
$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)} \quad \text{with } R = r'^2$$

In general, FP Interferometer can be realized with two parallel partially reflecting mirrors

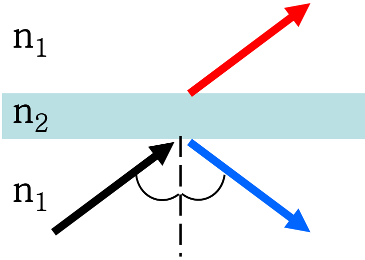
$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)} \quad \text{with } R = r^2$$

Lect. 8: Interferometers

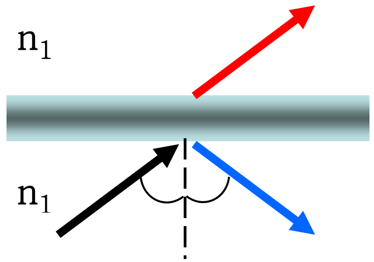
Michelson Interferometer: Two mirrors and one beam splitter



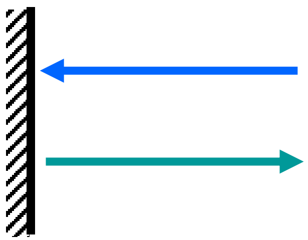
Frustrated TIR



Partially Coated Mirror

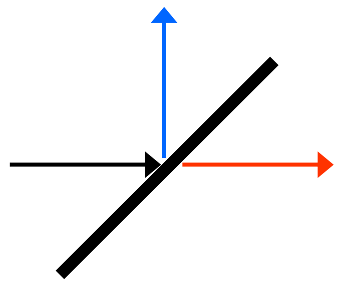


- Mirror (perfect conductor)



R=1, r=-1

- Beam Splitter

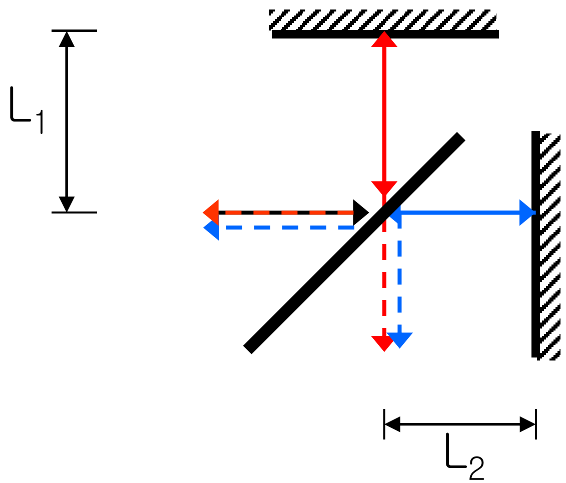


$$R = \frac{1}{2},\ T = \frac{1}{2}$$

$$r = ?,\ t = ?$$

Lect. 8: Interferometers

Michelson Interferometer:



Four outputs:

Side	1:	r	r
	2:	t	t
Bottom	1:	r	t
	2:	t	r

$$E_{out,\,side} = re^{-jkl_1}(-1)e^{-jkl_1}r + te^{-jkl_2}(-1)e^{-jkl_2}t = -r^2e^{-j2kl_1} - t^2e^{-j2kl_2}$$

$$E_{out,\,bottom} = re^{-jkl_1}(-1)e^{-jkl_1}t + te^{-jkl_2}(-1)e^{-jkl_2}r = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

Lect. 8: Interferometers

With $r = t = \dfrac{1}{\sqrt{2}}$,

$$E_{out,\,side} = -r^2 e^{-j2kl_1} - t^2 e^{-j2kl_2}$$

$$= -\frac{1}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right) = -\frac{1}{2} e^{-jk(l_1+l_2)}\left(\mathrm{e}^{-jk(l_1-l_2)} + \mathrm{e}^{jk(l_1-l_2)}\right)$$

$$I_{out,\,side} = \left|E_{out,\,side}\right|^2 = \cos^2[k(l_1 - l_2)]$$

$$E_{out,\,bottom} = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

$$= -\frac{1}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right)$$

$$I_{out,\,bottom} = \left|E_{out,\,bottom}\right|^2 = \cos^2[k(l_1 - l_2)]$$

Against energy conservation!

Lect. 8: Interferometers

With $r = \frac{1}{\sqrt{2}}, \ t = j\frac{1}{\sqrt{2}}$

(r and t should have $\pi/2$ phase difference in order to satisfy energy conservation)

$$E_{out,\,side} = -r^2 e^{-j2kl_1} - t^2 e^{-j2kl_2}$$

$$= -\frac{1}{2}\left(e^{-j2kl_1} - e^{-j2kl_2}\right) = -\frac{1}{2}e^{-jk(l_1+l_2)}\left(\mathrm{e}^{-jk(l_1-l_2)} - \mathrm{e}^{jk(l_1-l_2)}\right)$$

$$I_{out,\,side} = \left|E_{out,\,side}\right|^2 = \sin^2[k(l_1 - l_2)]$$

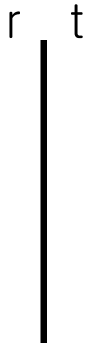
$$E_{out,\,bottom} = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

$$= -\frac{j}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right)$$

$$I_{out,\,bottom} = \left|E_{out,\,bottom}\right|^2 = \cos^2[k(l_1 - l_2)]$$

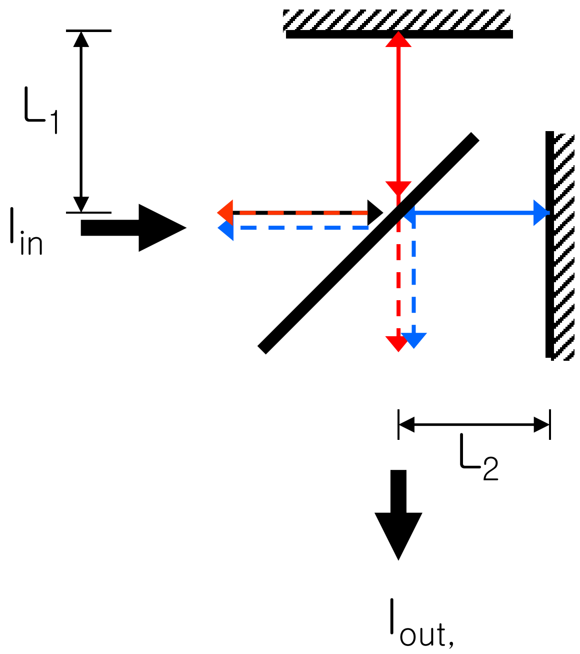
Lect. 8: Interferometers

Example: What is r and t if the mirror reflects 50% of incident light?



Lect. 8: Interferometers

Michelson Interferometer:



$$\frac{I_{out,bottom}}{I_{in}} = \cos^2[k(l_1 - l_2)]$$

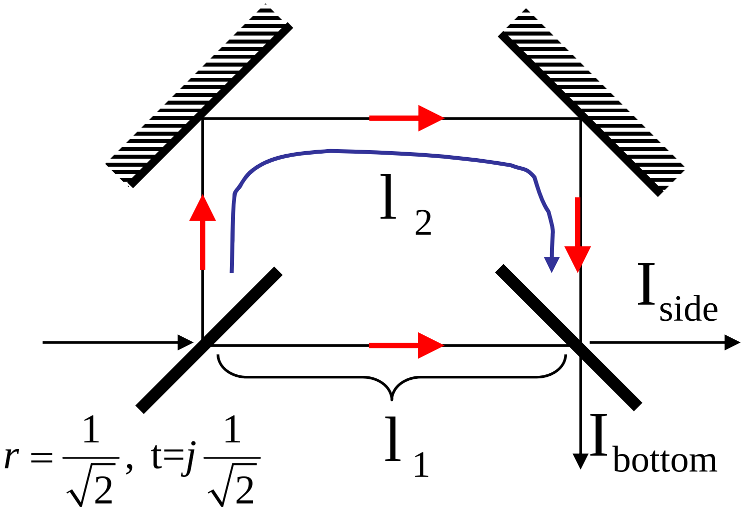
$k(l_1 - l_2)$ can be measured very precisely

- Experimental measurement of speed of light by Michelson in 1879.

➔ Provided a clue for Special Relativity:
speed of light is same always
(Michelson Morley experiment)

Lect. 8: Interferometers

Mach-Zehnder Interferometer:



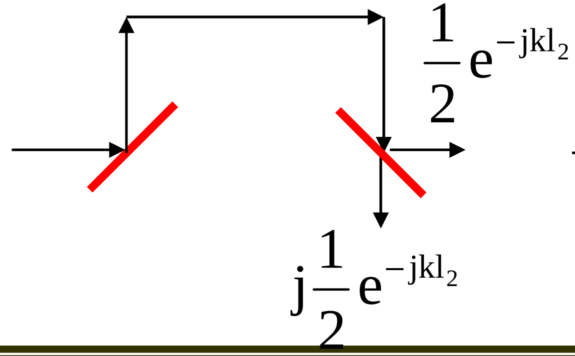
$$E_{out,\,side} = \frac{1}{2}\left(e^{-jkl_2} - e^{-j2kl_1}\right) = \frac{1}{2}e^{-jk\frac{l_2+l_1}{2}}\left(e^{-jk\frac{l_2-l_1}{2}} - e^{jk\frac{l_2-l_1}{2}}\right)$$

$$I_{out,\,side} = \sin^2\left(k\frac{l_1-l_2}{2}\right)$$

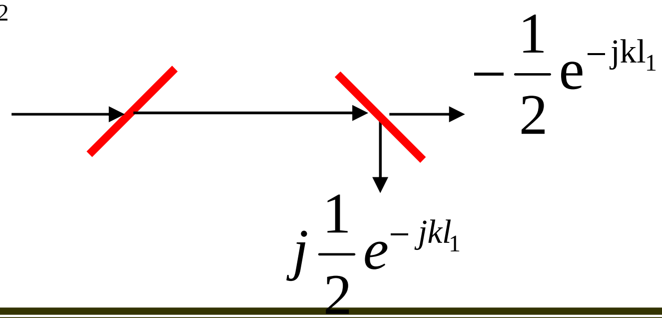
$$E_{out,\,bottom} = \frac{j}{2}\left(e^{-jkl_1} + e^{-jkl_2}\right) = \frac{j}{2}e^{-jk\frac{l_1+l_2}{2}}\left(e^{-jk\frac{l_1-l_2}{2}} + e^{jk\frac{l_1-l_2}{2}}\right)$$

$$I_{out,\,bottom} = \cos^2\left(k\frac{l_1-l_2}{2}\right)$$

<u>Case#1</u>



<u>Case#2</u>



Lect. 8: Interferometers

Can any EM wave cause interference?
Only EM waves having clear phase relationship cause interference: coherent

How large $(l_1 - l_2)$ can be?
Any long as two separated waves are coherent or within coherent length

Separated waves become incoherent due to intrinsic phase noises

Homework: Prob. 3 in 2001 Test #2
(Optional)